Titolo**: Time from Quantum Entanglement**

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In this Thesis we have reviewed and extended a theory in which time is an emergent property of quantum entaglement. The idea that time may emerge from quantum entanglement originated from a mechanism proposed in 1983 by Don Page and William Wootters to solve the, so called, "problem of time" that arises in the context of canonical quantization of gravity. According to the Wheeler-DeWitt equation the whole universe should be in an eigenfunction of the total Hamiltonian, so it would not show any dynamical behavior. The Page and Wootters (PaW) theory consists in dividing the total Hilbert space into two sub-systems and assigning one of it to time: we call these two systems the "clock" and the "rest". The flow of time then consists simply in the correlation (entanglement) between the quantum degree of freedom of time and the rest, a correlation present in a global, time-independent state. In this framework we do not consider time as an abstract, external coordinate, but as "what is shown on a clock", where the clock is some physical system that is taken as time reference. So, under this light, a very interesting question is to understand which systems can be chosen as clocks: which are the ideal systems and also which are the most physical. Up to now, to describe the Hilbert space of the clock subsystem, have been chosen a few special systems, therefore our work will primarily focus on which Hamiltonians can be used as clock Hamiltonians, and our intention is to generalize as much as possible the PaW mechanism to be able to use a large number of physical systems as a clock.

For this purpose in we start from a work of D. T. Pegg in which he explores the possibility of the existence of a quantity that can be regarded as the complement of the Hamiltonian for a quantum system with discrete energy levels. He examined this question for a system with discrete energy eigenstates for which the ratios of the energy differences are rational and he found that such a quantity (the α quantity) does exist and can be represented both by a POVM and by an Hermitian operator, but in a state space larger than the minimal space needed to include the states of the system. The α quantity has dimensions of time, but it is not time. Rather this quantity is a property of the system and its behavior profoundly depends on the state of the system. So Pegg suggests to describe such a quantity with the name "age".

Our basic idea is to show that the Pegg's formalism can be accomodating and find a physical justification within the Page and Wootters framework. On the one hand, using Pegg states as clock states in PaW mechanism, allow us to use a large set of Hamiltonians as clock Hamiltonian. The only constraint prevents us in using Hamiltonians with discrete energy eigenstates for which the ratios of the energy differences are not rational. On the other hand, in the particular case in which the clock's Hamiltonian is chosen to be equally spaced in energy spectrum, is possible to construct an Hermitian operator **T** that is the complement of the clock's Hamiltonian in the same space of the space that includes the states of the system. It is well known the Pauli objection regarding the existence of a time operator: time and energy must have the same spectrum since conjugate operator are unitarily equivalent, but this is clearly not always true because normal Hamiltonians have lower bounded spectrum. In the PaW mechanism this objection is overcome and it is possible considering the operator **T** as a time operator for each generic Hamiltonian of the rest of the system.

Finally, we will consider the mechanism in order to describe the spatial degree of freedom. Indeed it is possible to read the PaW approach as a sort of internalization of the temporal reference frame; time is no longer measured by an external clock (the external time) but is thought as "what is shown on a clock", where the clock is a quantum system correlated with the system under investigation. So, in order to generalize the PaW mechanism, we ask: is it possible to apply the same formalism to space? That is, is it possible to internalize the reference frame so that space can also be thought as "what is shown on a meter"? So, we try to extend the mechanism to space and prepare the basis for a possible simple approach to quantum spacetime.